RCA, PROBLEM SET 3

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0.1. Prove that the completion functor $M \mapsto M^{\wedge_0}$ is an equivalence $\mathcal{O}_c \to \mathcal{O}_c^{\wedge_0}$ with inverse $N \mapsto \bigoplus_\lambda N_\lambda$.

0.2. Show that KZ and KZ$'$ intertwine the functors ${}^{\mathcal{O}}\mathrm{Ind}_W^{W'}$, ${}^{\mathcal{H}}\mathrm{Ind}_W^{W'}$. For this show that the induction and restriction map between $\mathcal{O}_c(W)^{tor}, \mathcal{O}_c(W')^{tor}$.

0.3. Show that, for $M \in \mathcal{O}_c$, the following are equivalent:

- M is free over $\mathbb{C}[\mathfrak{h}]$,
- M is standardly filtered.

In this case, the class of M in K_0 coincides with $M/\mathfrak{h}M$.

Formulate and prove an analogous statement for $\mathcal{O}_c^{\wedge_0}$.

0.4. Show that ${}^{\mathcal{O}}\mathrm{Ind}_W^{W'}$ is exact. For this, you'll need to answer the questions: where do continuous duals of modules in $\mathcal{O}_c^{\wedge_{W_b}}$ live? how to describe $\mathrm{Ind}(\bullet^\vee)^\vee$.