LECTURES ON SYMPLECTIC REFLECTION ALGEBRAS

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14. Quantum Hamiltonian reduction and SRA for wreath-products

14.1. **Quantum comoment maps.** Let $\mathcal{A}_\hbar$ be an associative unital algebra over $\mathbb{C}[\hbar]$ that is flat and separated in the $\hbar$-adic topology and such that $\mathcal{A}_\hbar/(\hbar)$ is commutative. As we have seen in Lecture 12, we have a natural Poisson bracket on $A := \mathcal{A}_\hbar/(\hbar)$: it is induced by $\frac{1}{\hbar}[\cdot,\cdot]$.

We suppose that an algebraic group G acts on $\mathcal{A}_\hbar$ by $\mathbb{C}[\hbar]$-algebra automorphisms. We will use two different settings.

(S1) $\mathcal{A}_\hbar$ is complete in the $\hbar$-adic topology and the action of G is pro-rational, i.e., the induced action of G on every quotient $\mathcal{A}_\hbar/(\hbar^k)$ is rational.

(S2) $\mathcal{A}_\hbar$ is graded, $\mathcal{A}_\hbar = \bigoplus_{i=0}^{+\infty}\mathcal{A}_\hbar^i$, with $\hbar$ of some positive degree, say d, and G preserves the grading and acts rationally.

We remark that in the second case the Poisson bracket on A is of degree $-d$.

We will mostly use (S2) but still occasionally need (S1).

In both cases we have the induced representation of $\mathfrak{g}$ on $\mathcal{A}_\hbar$ and this representation is by derivations. Let $\xi_{\mathcal{A}}$ denote the derivation corresponding to ξ. Of course, the map $\xi\mapsto\xi_{\mathcal{A}}$ is G-equivariant.

By a quantum comoment map for the action of G on $\mathcal{A}_\hbar$ we mean a linear map $\Phi:\mathfrak{g}\to\mathcal{A}_\hbar$ that is G-equivariant and satisfies $\frac{1}{\hbar}[\Phi(\xi),\cdot]=\xi_{\mathcal{A}}$. We remark that a quantum comoment map is not recovered uniquely. For example, under the assumption that G is connected, if Φ is a quantum comoment map and Ψ is a map from $\mathfrak{g}/[\mathfrak{g},\mathfrak{g}]$ to the center of $\mathcal{A}_\hbar$, then $\Phi+\Psi$ is also a quantum comoment map and all quantum comoment maps are obtained in this way. Also we remark that modulo $\hbar$, the map Φ is a classical comoment map. Finally, in the setting (S2) we always assume that Φ lands in degree d.

Exercise 14.1. *Prove that* $\Phi([\xi,\eta])=\frac{1}{\hbar}[\Phi(\xi),\Phi(\eta)]$ *for any* $\xi,\eta\in\mathfrak{g}$.

Let us explain two (related) examples of quantum comoment maps. First, let V be a symplectic vector space and consider the Weyl algebra $W_\hbar(V)$ (with its usual grading, where the degree of $\hbar$ is 2). The group $G=\mathrm{Sp}(V)$ acts by automorphisms of $W_\hbar(V)$. We are going to establish a quantum comoment map for this action.

The degree 1 component $W_\hbar(V)^1$ is naturally identified with V. Now for $a\in W_\hbar(V)^2$ the map $\frac{1}{\hbar}[a,\cdot]:W_\hbar(V)\to W_\hbar(V)$ is degree preserving. In particular, $W_\hbar(V)^2$ is a Lie subalgebra with respect to the bracket $\frac{1}{\hbar}[\cdot,\cdot]$ and V is a module over this algebra. The symplectic form on V can also be described as $\frac{1}{\hbar}[\cdot,\cdot]$. From the Jacobi identity for $W_\hbar(V)$ applied to elements from $W_\hbar(V)^2, W_\hbar(V)^1, W_\hbar(V)^1$, we see that the action of $W_\hbar(V)^2$ on $W_\hbar(V)^1$ annihilates the symplectic form and so we get a Lie algebra homomorphism $W_\hbar(V)^2\to\mathfrak{sp}(V)$. Definitely, $\hbar\in W_\hbar(V)^2$ lies in the kernel. We claim that the kernel is spanned by this element. Indeed, any element of the kernel lies in the center of $W_\hbar(V)$ because that algebra is spanned by V.

Exercise 14.2. *Prove that the center of* $W_\hbar(V)$ *coincides with* $\mathbb{C}[\hbar]$.

As a vector space, $W_\hbar(V)^2 = S^2(V) \oplus \mathbb{C}\hbar$. So the dimensions of $W_\hbar(V)^2/\mathbb{C}\hbar$ and $\mathfrak{sp}(V)$ coincide. Therefore the homomorphism $W_\hbar(V)^2 \to \mathfrak{sp}(V)$ is surjective. So $W_\hbar(V)^2$ is an extension of $\mathfrak{sp}(V)$ by $\mathbb{C}$ that is forced to split (because $\mathfrak{sp}(V)$ is simple) and actually in a unique, and hence $\mathrm{Sp}(V)$-equivariant, way. For Φ we take this splitting, it is a quantum comoment map. The latter follows from the observation that ξ_W acts on $V = W_\hbar(V)^1$ as the operator ξ.

We remark that if G acts on V by linear symplectomorphisms, then the induced action of G on $W_\hbar(V)$ also admits a quantum comoment map, the composition of the induced homomorphism $\mathfrak{g} \to \mathfrak{sp}(V)$ with Φ constructed above.

Let us proceed to our second example. Let X_0 be a smooth affine variety acted on by an algebraic group G. Then we can form the algebra $D_\hbar(X_0)$ of homogenized differential operators. This algebra is graded ($\mathbb{C}[X_0]$ has degree 0, while $\hbar$ and $\mathrm{Vect}(X_0)$ have degree 1), and G satisfying the assumptions of (S2).

Exercise 14.3. *Describe the map $\xi \mapsto \xi_{\mathcal{A}}$ for $\mathcal{A}_\hbar = D_\hbar(X_0)$ and show that $\xi \mapsto \xi_{X_0}$ is a quantum comoment map.*

Now consider the special case when X_0 is a vector space. Then $D_\hbar(X_0)$ is naturally identified with $W_\hbar(X_0 \oplus X_0^*)$. We have two quantum comoment maps, Φ_W and Φ_D.

Problem 14.4. *Describe the difference $\Phi_D - \Phi_W$.*

In these examples we only used setting (S2). We can get setting (S1) if we pass to the $\hbar$-adic completions. This useful for the reason that many constructions from commutative algebra, like localization or completion, do not work with (S2) but do with (S1).

Exercise 14.5. *Let $\mathcal{A}_\hbar$ be an associative unital algebra over $\mathbb{C}[\hbar]$, flat over $\mathbb{C}[\hbar]$, complete and separated in the $\hbar$-adic topology, and such that $A := \mathcal{A}_\hbar/(\hbar)$ is commutative. Let S be a mutliplicatively closed subset of A that does not contain 0 and let π_k denote the projection $\mathcal{A}_\hbar/(\hbar^k) \twoheadrightarrow A$. Show that $\pi_k^{-1}(S)$ satisfies the Ore condition: i.e., for all $a \in \mathcal{A}_\hbar/(\hbar^k), s \in \pi_k^{-1}(S)$, there are $a' \in \mathcal{A}_\hbar/(\hbar^k), s' \in \pi_k^{-1}(S)$ such that $as' = a's$. Show that there are natural epimorphisms $\mathcal{A}_\hbar/(\hbar^{k+1})[\pi_{k+1}(S)^{-1}] \twoheadrightarrow \mathcal{A}_\hbar/(\hbar^k)[\pi_k(S)^{-1}]$ and prove that $\mathcal{A}_\hbar[S^{-1}] := \varprojlim_k \mathcal{A}_\hbar/(\hbar^k)[\pi_k(S)^{-1}]$ is flat over $\mathbb{C}[[\hbar]]$.*

14.2. **Quantum Hamiltonian reduction.** Let $\mathcal{A}_\hbar, G, \Phi$ be as in the previous section. We can consider the quantum Hamiltonian reduction $\mathcal{A}_\hbar///_0 G := [\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi(\mathfrak{g})]^G$. The latter space is an associative unital $\mathbb{C}[\hbar]$-algebra with product given by $(a+\mathcal{A}_\hbar\Phi(\mathfrak{g}))\cdot(b+\mathcal{A}_\hbar\Phi(\mathfrak{g})) = ab + \mathcal{A}_\hbar\Phi(\mathfrak{g})$. We remark that in setting (S2), this algebra is naturally graded, the grading is induced from $\mathcal{A}_\hbar$.

As in the Poisson case, this construction can be generalized to the reduction at ideals. Namely, thanks to Exercise 14.1, the map $\Phi : \mathfrak{g} \to A_\hbar$ extends to a G-equivariant (graded for (S2)) algebra homomorphism $U_\hbar(\mathfrak{g}) \to \mathcal{A}_\hbar$, where $U_\hbar(\mathfrak{g})$ is a homogenized universal enveloping algebra defined as follows

$$U_\hbar(\mathfrak{g}) = T(\mathfrak{g})[\hbar]/(\xi \otimes \eta - \eta \otimes \xi - [\xi, \eta]\hbar).$$

Here the grading on $U_\hbar(\mathfrak{g})$ is defined so that $\deg \mathfrak{g} = \deg \hbar = d$.

We pick a graded G-stable two-sided ideal $\mathcal{I} \subset U_\hbar(\mathfrak{g})$ that is $\hbar$-saturated in the sense that $\hbar x \in \mathcal{I}$ implies $x \in \mathcal{I}$ (equivalently, the quotient $U_\hbar(\mathfrak{g})/\mathcal{I}$ is flat over $\mathbb{C}[\hbar]$). Then we set $\mathcal{A}_\hbar///_{\mathcal{I}} G := [\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi(\mathcal{I})]^G$. This is an associative algebra with respect to a product analogous to the above.

Exercise 14.6. *Show that the product on $\mathcal{A}_\hbar///_{\mathcal{I}}G$ is well-defined.*

For example, for $\mathcal{I}$ we can take $\mathfrak{g}U_\hbar(\mathfrak{g})$ so that $U_\hbar(\mathfrak{g})/\mathcal{I} = \mathbb{C}[\hbar]$. Another option, for $\lambda \in \mathfrak{g}^{*G}$ we can consider the ideal in $U_\hbar(\mathfrak{g})$ generated by the ideal $\{\xi - \hbar\langle\lambda, \xi\rangle | \xi \in \mathfrak{g}\}$. The corresponding reduction will be denoted by $\mathcal{A}_\hbar///_{\lambda\hbar}G$. Finally, and this will be our favorite choice, we can consider the ideal $\mathcal{I} = [\mathfrak{g}, \mathfrak{g}]U_\hbar(\mathfrak{g})$. We have $U_\hbar(\mathfrak{g})/\mathcal{I} = S(\mathfrak{g}/[\mathfrak{g}, \mathfrak{g}])[\hbar]$. The corresponding reduction will be denoted by $\mathcal{A}_\hbar///G$. This is an algebra over $S(\mathfrak{g}/[\mathfrak{g}, \mathfrak{g}])[\hbar]$. A map $\mathfrak{g}/[\mathfrak{g}, \mathfrak{g}] \to \mathcal{A}_\hbar///G$ is induced by Φ (we mod out $[\mathfrak{g}, \mathfrak{g}]$).

Exercise 14.7. *Check that the image of $\mathfrak{g}/[\mathfrak{g}, \mathfrak{g}]$ in $[\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi([\mathfrak{g}, \mathfrak{g}])]$ consists of G-invariant elements that commute with $[\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi([\mathfrak{g}, \mathfrak{g}])]^G$.*

The reduction $\mathcal{A}_\hbar///_{\lambda\hbar}G$ is the specialization $\mathbb{C}[\hbar]\otimes_{S(\mathfrak{g}/[\mathfrak{g},\mathfrak{g}])[\hbar]}\mathcal{A}_\hbar///G$ for the homomorphism $S(\mathfrak{g}/[\mathfrak{g}, \mathfrak{g}])[\hbar] \to \mathbb{C}[\hbar]$ given by $\xi \mapsto \langle\lambda, \xi\rangle\hbar$.

Problem 14.8. *Let G be a reductive group acting freely on a smooth affine variety X_0. Identify $D_\hbar(X_0)///_0G$ with $D_\hbar(X_0//G)$.*

Below we will denote $\mathfrak{g}/[\mathfrak{g}, \mathfrak{g}]$ by $\mathfrak{z}$.

14.3. **Sufficient condition for flatness.** From now on, we assume that the group G is reductive, in particular, $\mathfrak{g} = \mathfrak{z} \oplus [\mathfrak{g}, \mathfrak{g}]$. Under this assumption, we have

$$[\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi([\mathfrak{g}, \mathfrak{g}])]^G/(\mathfrak{z}, \hbar) = [\mathcal{A}_\hbar/(\mathcal{A}_\hbar\Phi([\mathfrak{g}, \mathfrak{g}]) + (\mathfrak{z}, \hbar))]^G = [A/\mu^*(\mathfrak{g})A]^G = A///_0G$$

We want to find conditions for $\mathcal{A}_\hbar///G$ to be flat over $S(\mathfrak{z})[\hbar]$. We will assume that G is reductive and that $A = \mathcal{A}_\hbar/(\hbar)$ is a finitely generated algebra. Let X denote the corresponding scheme and let $\mu : X \to \mathfrak{g}^*$ be the moment map (that comes from the comoment map given by Φ modulo $\hbar$).

Proposition 14.1. *Let $\mathcal{A}_\hbar$ be as in (S2) and assume, in addition, that the grading is positive, i.e., $\mathcal{A}_\hbar^0 = \mathbb{C}$. Suppose that $\operatorname{codim}_X \mu^{-1}(0) = \dim \mathfrak{g}$. Then $\mathcal{A}_\hbar///G$ is flat over $S(\mathfrak{z})[\hbar]$.*

Proof. Recall that a sequence of elements $f_1, \ldots, f_k \in A$ is called *regular* if, for each i, the element f_i is not a zero divisor in $A/(f_1, \ldots, f_{i-1})$. This is equivalent to the condition that the subscheme defined by $f_1, \ldots, f_k$ has codimension k.

The proof of the proposition is based on the following property of regular sequences, see, for example, [E, Chapter 17]. Assume that A is a $\mathbb{Z}_{\geqslant 0}$-graded algebra with $A^0 = \mathbb{C}$. Suppose that $f_1, \ldots, f_k$ is a regular sequence of homogeneous elements and $g_1, \ldots, g_k \in A$ are such that $\sum_{i=1}^k f_i g_i = 0$. Then there are elements $g_{ij} \in A$ with $g_{ij} = -g_{ji}$ with the property that $g_i = \sum_{i=1}^k g_{ij} f_j$ (obviously, for this choice of the elements g_i the sum $\sum_{i=1}^k f_i g_i$ vanishes).

Let $x_1, \ldots, x_m$ be a basis in $[\mathfrak{g}, \mathfrak{g}]$, and $z_1, \ldots, z_k$ be a basis in $\mathfrak{z}$. First, we want to show that the ideal $A_\hbar\Phi([\mathfrak{g}, \mathfrak{g}])$ is $\hbar$-saturated. Let $a \in \mathcal{A}_\hbar$ be such that $\hbar a \in \mathcal{A}_\hbar\Phi([\mathfrak{g}, \mathfrak{g}])$. We need to check that $\hbar a \in \hbar\mathcal{A}_\hbar\Phi([\mathfrak{g}, \mathfrak{g}])$. We can write a as $\sum_{i=1}^m G_i\Phi(x_i)$, where we can assume that at least one G_i is not divisible by $\hbar$. Let g_i be the class of G_i modulo $\hbar$. We have $\sum_{i=1}^m g_i\mu^*(x_i) = 0$. Since $\mu^*(x_1), \ldots, \mu^*(x_m)$ form a regular sequence in A, we see that there are elements $g_{ij} \in A$ with $g_{ij} = -g_{ji}$ and $g_i = \sum_j g_{ij}\mu^*(x_j)$. Lift the elements g_{ij} to $G_{ij} \in \mathcal{A}_\hbar$ with $G_{ij} = -G_{ji}$. We deduce that $G_i = \sum_{j=1}^m G_{ij}\Phi(x_j) + \hbar G_i'$ for some $G_i' \in \mathcal{A}_\hbar$. We can rewrite the sum $\sum_{i=1}^m G_i\Phi(x_i)$ as

$$\sum_{i,j=1}^m G_{ij}\Phi(x_j)\Phi(x_i)+\hbar\sum_{i=1}^m G_i'\Phi(x_i)=\sum_{i<j} G_{ij}[\Phi(x_j),\Phi(x_i)]+\hbar\sum_{i=1}^m G_i'\Phi(x_i)=$$

$$\hbar\sum_{i<j} G_{ij}\Phi([x_j,x_i])+\hbar\sum_{i=1}^m G_i'\Phi(x_i).$$

This shows that $\hbar a\in \hbar\mathcal{A}_\hbar\Phi([\mathfrak{g},\mathfrak{g}])$.

So far, we have only used that $\mu^*(x_1),\ldots,\mu^*(x_m)$ form a regular sequence. Since $\mu^*(x_1),\ldots,\mu^*(x_m),\mu^*(z_1),\ldots,\mu^*(z_k)$ form a regular sequence in A, we see that $\mu^*(z_1),\ldots,\mu^*(z_k)$ form a regular sequence in $A/A\mu^*([\mathfrak{g},\mathfrak{g}])$. Therefore $\mu^*(z_i)$ is a nonzero divisor in

$$[A/A\mu^*([\mathfrak{g},\mathfrak{g}])]/(\mu^*(z_1),\ldots,\mu^*(z_{i-1})).$$

Now the claim of the proposition follows from the following general fact that is left as an exercise:

Let $M=\bigoplus_{i=0}^{+\infty}M_i$ with $\dim M_i<\infty$ be a graded module over the polynomial ring $\mathbb{C}[y_1,\ldots,y_l]$ (where all y_i's are supposed to have positive degrees). Then the following two conditions are equivalent:

(i) M is a graded free module.
(ii) y_i is a nonzero divisor in $M/(y_1,\ldots,y_{i-1})M$.

We apply this to $M=\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi([\mathfrak{g},\mathfrak{g}])$ and $y_1=\hbar, y_2=\mu^*(z_1),\ldots,y_l=\mu^*(z_k)$. □

In particular, if the assumption of Proposition 14.1 holds, then all reductions $\mathcal{A}///_{\lambda\hbar}G$ are deformations of $A///_0G$ over $\mathbb{C}[\hbar]$.

14.4. **Spherical SRA as quantum Hamiltonian reductions.** We have already seen some connections between spherical subalgebras in SRA and Hamiltonian reductions: in the cases when a group Γ was a Kleinian subgroup $\Gamma_1\subset \mathrm{SL}_2(\mathbb{C})$ and a symmetric group $\mathfrak{S}_n$ acting on the double $\mathbb{C}^{2n}$ of its permutation representation $\mathbb{C}^n$. The Hamiltonian reduction in both cases was of similar nature: a space R being reduced was the representation space $\mathrm{Rep}(Q,v)$ of some double quiver Q and a group G was the product of several general linear groups. More precisely, in the case of a Kleinian group, Q was the double McKay quiver, v was the indecomposable imaginary root δ, and G was $\mathrm{GL}(\delta)$. In the case of a symmetric group, Q has two vertices, 0 and ∞, two loops at 0 and two arrows between 0 and ∞ going in opposite directions. The dimension vector v in this case equals $n\epsilon_0+\epsilon_\infty$, where $\epsilon_0,\epsilon_\infty$ are coordinate vectors at the corresponding vertices. Finally, we took $G=\mathrm{GL}(n)$.

It is natural to expect that a connection should extend to the case of $\Gamma=\Gamma_n=\mathfrak{S}_n\ltimes\Gamma_1^n$.. This is indeed so. For Q we take the double Q of the following quiver: we take the (undoubled) McKay quiver with an additional vertex ∞ and an additional arrow $\infty\to 0$. We set $v=n\delta+\epsilon_\infty$ and for G take $\mathrm{GL}(n\delta)$.

Theorem 14.2 (Gan-Ginzburg, [GG]). *In the above notation, we have the following.*

(i) *The fiber $\mu^{-1}(0)$ is reduced and has codimension $\dim G$ in R.*
(ii) *There is a $\mathbb{C}^\times$-equivariant isomorphism of schemes $R///_0G\cong\mathbb{C}^{2n}/\Gamma_n$.*

The theorem will be proved in the next lecture.

Here the $\mathbb{C}^\times$-action on $\mathbb{C}^{2n}/\Gamma_n$ is induced from the dilations action on $\mathbb{C}^{2n}$ and the action on $R///_0G$ is induced from the dilations action on R.

Thanks to (i) and Proposition 14.1, $\mathrm{W}_\hbar(R)///G$ is a graded deformation of $\mathbb{C}[R///_0G]$ over $S(\mathfrak{z})[\hbar]$. The dimension of $\mathfrak{z}$ coincides with the number of irreducible Γ_1-modules (provided $n>1$). So $\dim \mathfrak{z}\oplus\mathbb{C}\hbar$ coincides with the dimension of the parameter space P of the universal SRA.

Theorem 14.3. *There is a graded algebra isomorphism $eHe\to \mathrm{W}_\hbar(V)///G$ that maps P to $\mathfrak{z}\oplus\mathbb{C}\hbar$ and $t\in P$ to $\hbar$.*

It is possible to write an explicit formula for the isomorphism $P\to\mathfrak{z}\oplus\mathbb{C}\hbar$, we may return to this in a subsequent lecture.

Here is a brief history of Theorem 14.3. It was first proved by Holland in the case of Kleinian groups (strictly speaking not for our Q but for the double of the McKay quiver, but this difference is not essential in this case). Then Etingof and Ginzburg proved a somewhat weaker version for the symmetric groups. This result was refined by Gan and Ginzburg. Then Oblomkov proved an analog of the Etingof-Ginzburg result for cyclic Γ_1. His result was refined by Gordon. Finally, Etingof, Gan, Ginzburg and Oblomkov gave a proof in the remaining cases.

An alternative proof was given by the author in [L] (the reader is referred to that paper for references). This is a proof that we are going to explain.

14.5. **Outline of proof.** A problem with studying deformations of $\mathbb{C}^{2n}/\Gamma_n$ is that this variety is not smooth. In particular, there seems to be no deformation of $\mathbb{C}^{2n}/\Gamma_n$ with a categorical universality property. However, and we have already seen this, it is possible to relate deformations of $\mathbb{C}^{2n}/\Gamma_n$ to deformations of something smooth, namely the smash-product $\mathbb{C}[\mathbb{C}^{2n}]\#\Gamma_n$: the deformation eHe of $\mathbb{C}^{2n}/\Gamma_n$, by the very definition, can be "lifted" to a deformation H of $\mathbb{C}[\mathbb{C}^{2n}]\#\Gamma_n$, which now has a universality property.

One can try to consider a purely algebro-geometric resolution of $\mathbb{C}^{2n}/\Gamma_n$ and ask about its deformations. We are very fortunate here: there is a (non-unique) *symplectic resolution* $\widetilde{\mathbb{C}^{2n}/\Gamma_n}$ of $\mathbb{C}^{2n}/\Gamma_n$ and it also can be obtained by a suitable version of Hamiltonian reduction. Thanks to this, we can lift $\mathrm{W}_\hbar(V)///G$ to a deformation of the resolution (that will be a sheaf, not a single algebra). This deformation will be, in fact, universal, but we will not need that.

Then one needs to relate the deformations of two different kind of resolutions, $\mathbb{C}[\mathbb{C}^{2n}]\#\Gamma_n$ and $\widetilde{\mathbb{C}^{2n}/\Gamma_n}$. This will be done using a so called *Procesi bundle*, a vector bundle on $\widetilde{\mathbb{C}^{2n}/\Gamma_n}$ whose endomorphisms are $\mathbb{C}[\mathbb{C}^{2n}]\#\Gamma_n$.

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