PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

11. CM system and Hamiltonian reduction

Exercise 11.1. *Let X_0 be a smooth algebraic variety equipped with a free action of a finite group Γ. Show that $T^*(X_0/\Gamma)$ is naturally identified with $(T^*X_0)/\Gamma$ (and that the identification intertwines the symplectic forms).*

Exercise 11.2. *Let $f_1, \ldots, f_m$ be functions on a symplectic variety X such that $\{f_i, f_j\} = 0$ for all i, j. Show that the dimension of the span of $d_x f_1, \ldots, d_x f_m$ has dimension not exceeding $\frac{1}{2} \dim X$.*

Exercise 11.3. *Show that the action of G on $\mu^{-1}(O)^{Reg}$ is free. Also check that $\operatorname{im} \iota$ intersects any orbit and that elements $\iota(p), \iota(p')$ are G-conjugate if and only if p and p' are $\mathfrak{S}_n$-conjugate.*

Problem 11.1. *Show that the action of G on $\mu^{-1}(O)$ is free.*

Exercise 11.4. [1] *Prove that a G-invariant ideal in $S(\mathfrak{g})$ is automatically Poisson. Also show that the converse is true provided G is connected.*

Exercise 11.5. *Equip the algebra $A///_I G$ with a natural Poisson bracket.*

[1] This exercise as well as the next one already appeared in PSet 10