PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

12. CM systems and quantum mechanics

Exercise 12.1. [1] *Show that the trajectories for* $H = \frac{1}{2}\operatorname{tr}(Y^2)$ *on* $R = T^* \operatorname{Mat}_n(\mathbb{C})$ *are of the form* $(X - tY, Y)$.

Problem 12.1. *Prove part (2) of the main theorem (integration of the CM system) in Lecture 11.*

Problem 12.2. *Check that the symplectic forms on* $\tilde{\mu}^{-1}(E)//\tilde{G}$ *and* $\mu^{-1}(O)//G$ *(see the notation in the lecture notes) are the same.*

Exercise 12.2. *Show that the algebra* $D_\hbar(X_0)$ *is a deformation of* $\mathbb{C}[T^*X_0]$ *compatible with the usual bracket there. Hint: how does the sheaf* $D_\hbar(X_0)$ *on* X_0 *behave under étale base changes?*

Exercise 12.3. *Let* $\mathcal{A}_\hbar$ *be a* $\mathbb{Z}_{\geqslant 0}$*-graded* $\mathbb{C}[\hbar]$*-algebra with* $\hbar$ *being of positive degree. Let* $\mathcal{A}'_\hbar$ *be the* $\hbar$*-adic completion of* $\mathcal{A}_\hbar$*. Explain how to recover* $\mathcal{A}_\hbar$ *back from* $\mathcal{A}'_\hbar$ *using some natural action of* $\mathbb{C}^\times$ *on* $\mathcal{A}'_\hbar$.

Exercise 12.4. *Let* X_0 *be a smooth affine variety acted freely by a finite group* Γ*. Equip* $D_\hbar(X_0)$ *with a natural* Γ*-action by* $\mathbb{C}[\hbar]$*-algebra automorphisms and then identify* $D_\hbar(X_0)^\Gamma$ *with* $D_\hbar(X_0/\Gamma)$.

[1] This exercise and the next problem already appeared in Pset 11