PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

14. Quantum Hamiltonian reduction vs SRA

Exercise 14.1. *Prove that* $\Phi([\xi,\eta]) = \frac{1}{\hbar}[\Phi(\xi),\Phi(\eta)]$ *for any* $\xi,\eta\in\mathfrak{g}$.

Exercise 14.2. *Prove that the center of* $W_\hbar(V)$ *coincides with* $\mathbb{C}[\hbar]$.

Exercise 14.3. *Describe the map* $\xi\mapsto\xi_{\mathcal{A}}$ *for* $\mathcal{A}_\hbar = D_\hbar(X_0)$ *and show that* $\xi\mapsto\xi_{X_0}$ *is a quantum comoment map.*

Problem 14.1. *Let* X_0 *be a vector space equipped with a linear action of a group* G. *Then* $W_\hbar(X_0\oplus X_0^*)$ *is the same algebra as* $D_\hbar(X_0)$. *We get two quantum comoments maps,* Φ_D,Φ_W *for the* G*-action on this algebra. Describe the difference* $\Phi_D-\Phi_W$.

Exercise 14.4. *Let* $\mathcal{A}_\hbar$ *be an associative unital algebra over* $\mathbb{C}[\hbar]$, *flat over* $\mathbb{C}[\hbar]$, *complete and separated in the* $\hbar$*-adic topology, and such that* $A:=\mathcal{A}_\hbar/(\hbar)$ *is commutative. Let* S *be a mutliplicatively closed subset of* A *that does not contain* 0 *and let* π_k *denote the projection* $\mathcal{A}_\hbar/(\hbar^k)\twoheadrightarrow A$. *Show that* $\pi_k^{-1}(S)$ *satisfies the Ore condition: i.e., for all* $a\in\mathcal{A}_\hbar/(\hbar^k), s\in\pi_k^{-1}(S)$, *there are* $a'\in\mathcal{A}_\hbar/(\hbar^k), s'\in\pi_k^{-1}(S)$ *such that* $as'=a's$. *Show that there are natural epimorphisms* $\mathcal{A}_\hbar/(\hbar^{k+1})[\pi_{k+1}(S)^{-1}]\twoheadrightarrow\mathcal{A}_\hbar/(\hbar^k)[\pi_k(S)^{-1}]$ *and prove that* $\mathcal{A}_\hbar[S^{-1}]:=\varprojlim_k\mathcal{A}_\hbar/(\hbar^k)[\pi_k(S)^{-1}]$ *is flat over* $\mathbb{C}[[\hbar]]$.

Exercise 14.5. *Show that the product on* $\mathcal{A}_\hbar///_{\mathcal{I}}G$ *is well-defined.*

Exercise 14.6. *Check that the image of* $\mathfrak{g}/[\mathfrak{g},\mathfrak{g}]$ *in* $[\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi([\mathfrak{g},\mathfrak{g}])]$ *consists of* G*-invariant elements that commute with* $[\mathcal{A}_\hbar/\mathcal{A}_\hbar\Phi([\mathfrak{g},\mathfrak{g}])]^G$.

Problem 14.2. *Let* G *be a reductive group acting freely on a smooth affine variety* X_0. *Identify* $D_\hbar(X_0)///_0G$ *with* $D_\hbar(X_0//G)$.