PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

19. KZ functor I

Problem 19.1. *Let M_1, M_2 be D_X-modules that are coherent sheaves. Show that*

$$\dim \operatorname{Hom}_{D_X}(M_1, M_2) < \infty.$$

Exercise 19.1. [1] *Let M be an H_c-module with locally nilpotent action of $\mathfrak{h}$. Show that M is finitely generated iff the action of h on M is locally finite and all generalized eigen-subspaces are finite dimensional.*

Problem 19.2. *Show that $\operatorname{Ext}^i(\Delta(E), \nabla(E')) = \mathbb{C}$ if $E = E', i = 0$, and 0 else. Moreover, show that if $\operatorname{Ext}^1(\Delta(E), M) = 0$ for all E, then M is ∇-filtered, i.e., admits a filtration with successive quotients $\nabla(E')$.*

Problem 19.3. *A Δ-filtered object M is projective iff $\operatorname{Ext}^1(M, \Delta(E)) = 0$ for all E.*

Problem 19.4.　　(1) *Show that the double centralizer property is equivalent to $\operatorname{Ext}^1(M, P) = 0$ for any projective P and $M \in \mathcal{O}^{tor}$.*
　(2) *Use the naive duality to show that π is fully faithful on injectives.*
　(3) *Show that π has left adjoint $\pi^!$ and that $\pi \circ \pi^!$ is the identity on the image of π.*

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[1]This exercise and the next problem also appeared last time.