PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

2. CBH algebras (left from last time)

Exercise 2.5. *Prove that there are no non-constant invariant polynomials for the action of the one-dimensional torus* $\mathbb{C}^\times$ *on* $\mathbb{C}^n$ *given by* $t.(x_1, \ldots, x_n) = (tx_1, \ldots, tx_n)$.

Exercise 2.6. *Use the theorem (the only statement called this way in the lecture) to show that the closure of any orbit of a reductive group action on an affine variety contains a unique closed orbit.*

Problem 2.6. *Show that the algebra of invariants* $\mathbb{C}[X]^G$*, where* $X = \mathrm{Mat}_n(\mathbb{C})$ *and* $G = \mathrm{GL}_n(\mathbb{C})$ *acts on* X *by conjugations, is generated by the coefficients of the characteristic polynomial of a matrix and is isomorphic to the algebra of polynomials in* n *variables. A hint: consider the restriction to the subspace of diagonal matrices.*

Problem 2.7. *In the setting of the previous problem, check directly that every fiber indeed contains a single closed orbit and that this orbit consists of diagonalizable matrices.*

3. McKay correspondence upgraded

Exercise 3.1. *Show that if the* $\mathrm{GL}_N(\mathbb{C})$*-orbit of a representation in* $\mathrm{Rep}(\mathcal{A}, N)$ *is closed, then the representation is semisimple.*

Problem 3.1. *Use the Hilbert-Mumford criterium to show that the orbit of a semisimple representation is closed.*

Exercise 3.2. *Show that the stabilizer in* Γ *of any nonzero point in* $\mathbb{C}^2$ *is trivial.*

Exercise 3.3. *A map* $\mathbb{C}^2 \otimes \mathbb{C}\Gamma \to \mathbb{C}\Gamma$ *extends to a representation from* $\mathrm{Rep}_\Gamma(\mathbb{C}\langle x, y\rangle \# \Gamma, \mathbb{C}\Gamma)$ *if and only if it is* Γ*-equivariant.*

Exercise 3.4. *Set* $M_{ij} := \mathrm{Hom}_\Gamma(\mathbb{C}^2 \otimes \mathbb{C}\Gamma, \mathbb{C}\Gamma)$. *Show that*

$$\mathrm{Hom}_\Gamma(\mathbb{C}^2 \otimes \mathbb{C}\Gamma, \mathbb{C}\Gamma) = \bigoplus_{i,j=0}^{r} M_{ij} \otimes \mathrm{Hom}_\mathbb{C}(N_i^*, N_j^*) = \bigoplus_{i,j=0}^{r} \mathrm{Hom}_\mathbb{C}(\mathbb{C}^{\delta_i}, \mathbb{C}^{\delta_j})^{m_{ij}}.$$