PROBLEMS ON SYMPLECTIC REFLECTION ALGEBRAS

3. McKay correspondence upgraded (from last time)

Exercise 3.3. *A map* $\mathbb{C}^2 \otimes \mathbb{C}\Gamma \to \mathbb{C}\Gamma$ *extends to a representation from* $\operatorname{Rep}_\Gamma(\mathbb{C}\langle x, y\rangle \# \Gamma, \mathbb{C}\Gamma)$ *if and only if it is* Γ*-equivariant.*

Exercise 3.4. *Show that*

$$\operatorname{Hom}_\Gamma(\mathbb{C}^2 \otimes \mathbb{C}\Gamma, \mathbb{C}\Gamma) = \bigoplus_{i,j=0}^{r} M_{ij} \otimes \operatorname{Hom}_{\mathbb{C}}(N_i^*, N_j^*)$$

$$= \bigoplus_{i,j=0}^{r} \operatorname{Hom}_{\mathbb{C}}(N_i^*, N_j^*)^{\oplus m_{ij}} = \bigoplus_{i,j=0}^{r} \operatorname{Hom}_{\mathbb{C}}(\mathbb{C}^{\delta_i}, \mathbb{C}^{\delta_j})^{m_{ij}}.$$

Note that the first equality is canonical, the second depends on the choice of a basis in M_{ij}*, while the third depends on the choice of bases in* N_i^**.*

4. Deformed preprojective algebras

Exercise 4.1. *Show that* $\mathbb{C}Q$ *is associative and* $\sum_{i\in Q_0} \epsilon_i$ *is a unit in* $\mathbb{C}Q$*. Further, show that, as a unital associative algebra,* $\mathbb{C}Q$ *is generated by* $\epsilon_i, i \in Q_0$*, and* $a \in Q_1$ *subject to the relations* $\epsilon_i\epsilon_j = \delta_{ij}\epsilon_i, \sum_{i\in Q_0} \epsilon_i = 1, \epsilon_i a = \delta_{ih(a)}a, a\epsilon_i = \delta_{it(a)}a$*.*

Exercise 4.2. *Use the universal properties of all algebras involved to show that* $\mathbb{C}\langle x, y\rangle \# \Gamma \cong T_{\mathbb{C}\Gamma}(\mathbb{C}^2 \otimes \mathbb{C}\Gamma)$ *and* $\mathbb{C}Q \cong T_{(\mathbb{C}Q)^0}(\mathbb{C}Q)^1$*.*

Exercise 4.3. *Let* A *be an associative algebra, and* $e \in A$ *be an idempotent. We define functors* $\pi : A\text{-Mod} \to eAe\text{-Mod}$ *by* $\pi(M) = eM$*, and* $\pi^! : eAe\text{-Mod} \to A\text{-Mod}$ *by* $\pi^!(N) = Ae \otimes_{eAe} N$*.*

- *Show that* π *is an exact functor, that* π *can be written as* $M \mapsto eA \otimes_A M$*, and that* $\pi^!$ *is left adjoint to* π*.*
- *Suppose that* $AeA = A$*. Check the that if* $\pi(M) = 0$*, then* $M = 0$*. Further check that the natural homomorphism* $Ae \otimes_{eAe} eM \to M$ *is surjective. Finally, show that* $Ae \otimes_{eAe} eM \to M$ *is injective by applying* π*.*
- *Deduce that* $Ae \otimes_{eAe} eA = A$ *as a bimodule.*

Exercise 4.4. *Suppose* e *is an idempotent in* A *such that* $AeA = A$*. Show that the functor* $M \mapsto eMe$ *is an equivalence between the categories of* A *and* eAe*-bimodules intertwining the tensor products (meaning that* $e(M \otimes_A N)e = eMe \otimes_{eAe} eNe$*). Deduce that* $eT_A(M)e$ *is naturally identified with* $T_{eAe}(eMe)$*.*

Exercise 4.5. *Check that the maps* $\operatorname{Hom}(M, \mathbb{C}^2 \otimes M') \to \operatorname{Hom}(\mathbb{C}^2 \otimes M, M'), \psi \mapsto (\omega \otimes 1_M) \circ (1_{\mathbb{C}^2} \otimes \psi)$ *and* $\operatorname{Hom}(\mathbb{C}^2 \otimes M, M') \to \operatorname{Hom}(M, \mathbb{C}^2 \otimes M'), \varphi \mapsto (1_{\mathbb{C}^2} \otimes \varphi) \circ (\zeta \otimes 1_M)$ *are inverse to each other.*

Problem 4.1. *Prove the CBH lemma in the cyclic case, assuming that the orientation on* $\underline{Q}$ *is also cyclic. Hint: for* x, y *we can take* Γ*-eigenvectors.*